Branching ratios of $B_s \to (K^+K^-, K^0\bar{K}^0)$ and $B_d \to \pi^+\pi^-$ and determination of $\gamma(\phi_3)$

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Abstract

We explored various cases for the branching ratios (BRs) of $B_s \to K^+K^-$, $B_s \to K^0\bar{K}^0$ and $B_d \to \pi^+\pi^-$ decays. We study the possibility of determining γ by using the following the measurements: (a) BRs of $B_s \to K^+K^-$, and $B_s \to K^0\bar{K}^0$; (b) the ratio of direct CP asymmetries in $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$; (c) the mix-induced CP asymmetry in $B_d \to \pi^+\pi^-$; and (d) the angle of β.

One of the purposes in present and future B physics experiments is to determine the CP violating angles of the Cabibbo-Kobayashi-Maskawa (CKM) matrix [1] induced from the three-generation quark mixings and described by the three angles α, β and γ or ϕ_2, ϕ_1 and ϕ_3. With the unitary property, they satisfy the triangular identity of $\alpha+\beta+\gamma=\pi$. In the literature, the time-dependent rate asymmetry of the $B_d \to \pi^+\pi^-$ decay is suggested to determine the angle α. However, large uncertainties from the inevitable pollution of the penguin-topology make this procedure be limited in experiments although some methods have been suggested to rescue them [2, 3]. Nevertheless, the similar approach applied to $B \to J/\Psi K_s$ for extracting β [4] is clean both theoretically and experimentally. The angle γ determination by the branching ratios (BRs) and direct CP asymmetries of $B \to K\pi$ decays has been also proposed [6, 7].

It is known that in the SU(3) flavor symmetry limit, there are relations between different decay amplitudes [8]. In particular, one has

$$\begin{aligned} \mathcal{A}(B_s \to K^0\bar{K}^0) &= \frac{V_t}{\lambda_t}\mathcal{A}(B_d \to K^0\bar{K}^0), \\ \mathcal{A}(B_s \to K^+K^-) + \mathcal{A}(B_s \to K^0\bar{K}^0) &= \frac{V_u}{\lambda_u}\left(\mathcal{A}\left(B_d \to \pi^+\pi^-\right) + \mathcal{A}\left(B_d \to K^0\bar{K}^0\right)\right). \end{aligned} \tag{1}$$

Furthermore, if the effects from annihilation contributions are negligible, we have new relations

$$\begin{aligned} \mathcal{A}(B_s \to K^+K^-) &= \mathcal{A}(B_d \to K^+\pi^-)\,, \\ \mathcal{A}(B_s \to K^0\bar{K}^0) &= \mathcal{A}(B_u^- \to K^0\pi^-)\,. \end{aligned} \tag{2}$$

Especially, except the different CKM matrix elements, under the U-spin transformation, since the subgroup of SU(3) describes the interchange of d- and s-quark, the transition matrix elements associated with various topologies in $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ decays are related to each other. By using the connected relations and combining the mix-induced and direct CP asymmetries in both $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ decays, Fleischer in Ref. [9] has proposed strategies to determine γ.

In this paper, we will first evaluate the branching ratios (BRs) of $B_s \to (K^+K^-,\ K^0\bar{K}^0)$ and $B_d \to \pi^+\pi^-$ and then analyze the possibility of extracting angle γ through $BR(B_s \to KK)$ and the relevant measurements in $B_d \to \pi^+\pi^-$. Other approaches associated with $B_s \to KK$ can refer to [10, 11, 12, 13]

We start by writing the amplitudes for $B_s \to K^+K^-$, $B_s \to K^0\bar{K}^0$, $B_d \to \pi^+\pi^-$ and $B_d \to K^0\bar{K}^0$ decays generally as

$$\begin{aligned} \mathcal{A}(B_s \to K^+K^-) &= V_tP_t + V_cP_c + V_u\left(P_u + T\right) \\ &= V_cP_{ct}\left(1 + \left|\frac{V_u}{V_c}\right| Re^{i(\delta+\gamma)}\right) \end{aligned} \tag{3}$$

$$\mathcal{A}(B_s \to K^0\bar{K}^0) = V_cP_{ct}\left(1 + \frac{V_u}{V_c}\frac{P_{ut}}{P_{ct}}\right) \tag{4}$$

$$\begin{aligned} \mathcal{A}(B_d \to \pi^+\pi^-) &= \lambda_tP'_t + \lambda_cP'_c + \lambda_u\left(P'_u + T'\right) \\ &= \lambda_cP'_{ct}\left(1 - \left|\frac{\lambda_u}{\lambda_c}\right| re^{i(\delta'+\gamma)}\right) \end{aligned} \tag{5}$$

$$\mathcal{A}(B_d \to K^0\bar{K}^0) = \lambda_cP''_{ct}\left(1 + \frac{\lambda_u}{\lambda_c}\frac{P''_{ut}}{P''_{ct}}\right) \tag{6}$$

where $P_q^{(\prime,\prime\prime)}$ and $T^{(\prime)}$ denote the penguin- and tree-topology contributions, $P_{qq'}^{(\prime,\prime\prime)} = P_q^{(\prime,\prime\prime)} - P_{q'}^{(\prime,\prime\prime)}$, $R(r)e^{i\delta^{(\prime)}} = \left(P_{ut}^{(\prime)} + T^{(\prime)}\right)/P_{ct}^{(\prime)}$, $\lambda_i = V_{id}^*V_{ib}$, $V_i = V_{is}^*V_{ib}$ in which V_{ij} are the CKM matrix

elements and they satisfy $\sum_i V_i(\lambda_i) = 0$. With the Wolfenstein's parametrization [14], we know that

$$\begin{array}{lll} \lambda_t = A\lambda^3 R_t e^{-i\beta}, & \lambda_c = -A\lambda^3, & \lambda_u = A\lambda^3 R_b e^{-i\gamma}, \\ V_t = -A\lambda^2, & V_c = A\lambda^2, & V_u = A\lambda^4 R_b e^{-i\gamma} \end{array}$$

where

$$\lambda \approx 0.22,\ A \approx 0.80,\ R_b \approx 0.36\,.$$

In the SU(3) limit, one finds that

$$\begin{aligned} P_{ct} &= P'_{ct} = P''_{ct}\,, \\ \frac{P_{ut}}{P_{ct}} &= \frac{P''_{ut}}{P''_{ct}}\,, \\ Re^{i\delta} &= re^{i\delta'}. \end{aligned} \tag{7}$$

However, by including SU(3)-breaking effects, the first relation in Eq. (7) becomes

$$\frac{P_{ct}}{f_K F^{B_s K}(0)} \simeq \frac{P'_{ct}}{f_\pi F^{B_d \pi}(0)} \simeq \frac{P''_{ct}}{f_K F^{B_d K}(0)}, \tag{8}$$

while the other two hold approximately, where we have set the light meson masses to be zero, i.e., $M_K^2 = 0$ and $M_\pi^2 = 0$, and f_P and $F^{B_q P}(0)$ denote the P meson decay constant and $B_q \to P$ decay form factor, respectively. Note that for parametrizing the SU(3) broken effects in Eq. (8), we have used the concept of the factorization assumption. In our following numerical analysis, we take $f_\pi = 0.13$, $f_K = 0.16$, $f_{B_d} = 0.19$, $f_{B_s} = 0.20$, $F^{B_d\pi} = 0.3$ [22], $F^{B_d K} = 0.35$ [21], and $F^{B_s K} = 0.33\ GeV$ [23]. Since the last two relations in Eq. (7) are the ratios of the transition matrix elements, the nonperturbative QCD effects can be reduced [9]. In order to estimate $|P_{ut}|\,/\,|P_{ct}|$, we use $\delta = 220^0$, $\gamma = 76^0$, $r = 8.0$ [9] and $Br\,(B_d \to \pi^+\pi^-) \simeq 5.43 \times 10^{-6}$ [15, 16, 17], then we get $|P'_{ct}|^2 \approx 1.54 \times 10^{-5}$. Using the value of $|P'_{ct}|$ and Eqs. (3) and (8), we obtain

$$Br\left(B_s \to K^+K^-\right) \simeq 20.83 \times 10^{-6}\,. \tag{9}$$

The result is consistent with that from Eq. (2), given by [8, 9]

$$\begin{aligned} Br\left(B_s \to K^+K^-\right) &\simeq \frac{\tau_{B_s}}{\tau_{B_d}} \left(\frac{M_{B_s}}{M_{B_d}}\right)^3 \left(\frac{F^{B_s K}(0)}{F^{B_d\pi}(0)}\right)^2 Br\left(B_d \to K^\pm\pi^\mp\right), \\ &\simeq 22.31 \times 10^{-6}\,, \end{aligned} \tag{10}$$

in which the measurement of $Br\,(B_d \to K^\pm\pi^\mp) \simeq 18.2 \times 10^{-6}$ [15, 16, 17] is used. From Eqs. (6) and (8) and the obtained value of $|P'_{ct}|$, the BR of $B_d \to K^0\bar{K}^0$ is given by

$$\begin{aligned} Br\left(B_d \to K^0\bar{K}^0\right) &= \tau_{B_d}\frac{G_F^2 M_{B_d}^3}{32\pi}\,|\lambda_c|^2\,|P''_{ct}|^2 \left(1 + R_b^2\left|\frac{P''_{ut}}{P''_{ct}}\right|^2 - 2R_b\left|\frac{P''_{ut}}{P''_{ct}}\right|\cos\gamma\cos\theta\right), \\ &= 1.11 \times 10^{-6}\left(1 + R_b^2\left|\frac{P''_{ut}}{P''_{ct}}\right|^2 - 2R_b\left|\frac{P''_{ut}}{P''_{ct}}\right|\cos\gamma\cos\theta\right) \end{aligned} \tag{11}$$

where the θ angle stands for the strong phase of P''_{ut}/P''_{ct}. It is clear that the upper bound on $|P''_{ut}/P''_{ct}|$ occurs to $\cos\gamma\cos\theta > 0$. By taking $\theta = 0^0$ and the limit of $Br\left(B_d \to K^0\bar{K}^0\right) \approx$

$Br\left(B^{\pm}\rightarrow K^{\pm}K^{0}\right)<2.5\times10^{-6}$ [8, 15], we get $|P''_{ut}/P''_{ct}|<4$ and $|P''_{ut}/P''_{ct}|=1$ if excluding rescattering effects. Hence, the second term in Eq. (4) can be neglected since $|V_u|/|V_c|\,|P''_{ut}/P''_{ct}|\simeq |V_u|/|V_c|\,|P_{ut}/P_{ct}|<0.069$, and Eq. (3) can be rewritten as

$$\mathcal{A}(B_s\rightarrow K^+K^-)=\mathcal{A}(B_s\rightarrow K^0\bar{K}^0)\left(1+\left|\frac{V_u}{V_c}\right|Re^{i(\delta+\gamma)}\right). \tag{12}$$

The CP averaged BR is given by

$$\begin{aligned}\bar{B}r\left(B_s\rightarrow K^+K^-\right) &\equiv \frac{Br\left(B_s\rightarrow K^+K^-\right)+Br\left(\bar{B}_s\rightarrow K^-K^+\right)}{2}\\ &= Br\left(B_s\rightarrow K^0\bar{K}^0\right)\left(1+\left|\frac{V_u}{V_c}\right|^2R^2+2\left|\frac{V_u}{V_c}\right|R\cos\gamma\cos\delta\right).\end{aligned} \tag{13}$$

From Eqs. (3–5) and (8), we now know that there are four unknown parameters, P_{ct}, r, δ' and γ in $B_s\rightarrow KK$ and $B_d\rightarrow\pi^+\pi^-$ decays. In terms of the analysis early, we see that $|P_{ct}|$ can be fixed by the measurement of $Br\left(B_s\rightarrow K^0\bar{K}^0\right)$. As shown in Ref. [9], the strong phase of δ' can be expressed as a function of r and γ by using the mix-induced CP asymmetry from the time-dependent decaying rate difference [5], defined by $a_{CP}(t)\equiv\left(\Gamma(B_d\rightarrow\pi^+\pi^-)-\Gamma(B_d\rightarrow\pi^+\pi^-)\right)/2$, and explicitly, one has

$$\begin{aligned}A_{CP}^{mix}\left(B_d\rightarrow\pi^+\pi^-\right) &= Im\left(e^{-i\phi_d}\frac{\bar{A}\left(B_d\rightarrow\pi^+\pi^-\right)}{A\left(B_d\rightarrow\pi^+\pi^-\right)}\right),\\ &= \frac{sin\phi_d-2\xi\cos\delta' sin\left(\phi_d+\gamma\right)+\xi^2 sin\left(\phi_d+2\gamma\right)}{1-2\xi\cos\delta'\cos\gamma+\xi^2}.\end{aligned} \tag{14}$$

From the above equation, we easily obtain

$$2\xi cos\delta'=\xi^2\rho+\omega \tag{15}$$

with

$$\begin{aligned}\rho &= \frac{A_{CP}^{mix}\left(B_d\rightarrow\pi^+\pi^-\right)-\sin\left(\phi_d+2\gamma\right)}{A_{CP}^{mix}\left(B_d\rightarrow\pi^+\pi^-\right)\cos\gamma-\sin\left(\phi_d+\gamma\right)},\\ \omega &= \frac{A_{CP}^{mix}\left(B_d\rightarrow\pi^+\pi^-\right)-\sin\phi_d}{A_{CP}^{mix}\left(B_d\rightarrow\pi^+\pi^-\right)\cos\gamma-\sin\left(\phi_d+\gamma\right)}\end{aligned}$$

where $\xi=|\lambda_u/\lambda_c|\,r$ and $\phi_d=2\beta$ comes from the $B_d-\bar{B}_d$ mixing and its present status in various experiments is listed as follows [18, 19, 20]:

$$\begin{aligned}sin\phi_d &= 0.58^{+0.32+0.09}_{-0.34-0.10} \qquad (\text{Belle}),\\ &= 0.59\pm0.14\pm0.05 \qquad (\text{BABAR}),\\ &= 0.79^{+0.41}_{-0.44} \qquad (\text{CDF}).\end{aligned} \tag{16}$$

In order to find the relationship between r and γ and fix them, one can use the direct CP asymmetry in $B_q\rightarrow PP$, defined by

$$A_{CP}^{dir}\left(B_q\rightarrow PP\right)=\frac{\Gamma\left(B_q\rightarrow PP\right)-\Gamma\left(\bar{B}_q\rightarrow PP\right)}{\Gamma\left(B_q\rightarrow PP\right)+\Gamma\left(\bar{B}_q\rightarrow PP\right)}, \tag{17}$$

where B_q can be B_d or B_s while P is π or K. From Eq. (15), $A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)$ and the ratio of $A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)$ to $A_{CP}^{dir}\left(B_s \to K^+K^-\right)$ [9], one has

$$\xi = \sqrt{\frac{1}{h}\left(l \pm \sqrt{l^2 - hk}\right)}, \tag{18}$$

and

$$\xi = \sqrt{\frac{1 - tR_{CP} + t(1+R_{CP})\omega\cos\gamma}{t(R_{CP}-t) - t(1+R_{CP})\rho\cos\gamma}}, \tag{19}$$

respectively, where

$$\begin{aligned}
\xi &= \left|\lambda_u/\lambda_c\right| r \\
h &= \rho^2 + C\left(1 - \rho cos\gamma\right)^2, \\
k &= \omega^2 + C\left(1 - \omega cos\gamma\right)^2, \\
l &= 2 - \rho\omega - C\left(1 - \rho cos\gamma\right)\left(1 - \omega cos\gamma\right), \\
C &= \left(\frac{A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)}{sin\gamma}\right)^2, \\
R_{CP} &= -\frac{A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)}{A_{CP}^{dir}\left(B_s \to K^+K^-\right)}, \\
t &= \left|\frac{V_u}{V_c}\right|\left|\frac{\lambda_c}{\lambda_u}\right|
\end{aligned}$$

In Figure 1, we show ξ as a function of γ in terms of Eqs. (18) and (19) with $A_{CP}^{mix}\left(B_d \to \pi^+\pi^-\right) = 0.45$, $A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right) = -0.23$, $R_{CP} = 1.4$ and $\sin 2\beta = 0.60$. From the figure, we see that the crossing points between Eqs. (18) and (19) are not unique. That is, it cannot completely settle down the r and γ with only the measurements of $A_{CP}^{mix}\left(B_d \to \pi^+\pi^-\right)$, $A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)$, R_{CP} and β. Therefore, one has to find another independent relation to fix them.

To do this, we use both BRs of $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ decays instead of using the time-dependent CP asymmetry for the $B_s \to K^+K^-$ decay of the approach in Ref. [9]. From Eqs. (13) and (15), we have

$$\xi = \sqrt{\frac{\left|\frac{\lambda_u}{\lambda c}\right|^2\left(R_B - 1\right) - \left|\frac{V_u}{V_c}\right|\left|\frac{\lambda_u}{\lambda c}\right|\omega cos\gamma}{\left|\frac{V_u}{V_c}\right|^2 + \left|\frac{V_u}{V_c}\right|\left|\frac{\lambda_u}{\lambda_c}\right|\rho cos\gamma}} \tag{20}$$

where

$$R_B = \frac{\bar{B}r\left(B_s \to K^+K^-\right)}{Br\left(B_s \to K^0\bar{K}^0\right)}.$$

In Eq. (10), we have obtained the BR of $B_s \to K^+K^-$ in a model independent way. Similarly, we have

$$\begin{aligned}
Br\left(B_s \to K^0\bar{K}^0\right) &\simeq \frac{\tau_{B_s}}{\tau_{B_d}}\left(\frac{M_{B_s}}{M_{B_d}}\right)^3\left|\frac{V_t}{\lambda_t}\frac{F^{B_sK}(0)}{F^{B_dK}(0)}\right|^2 Br\left(B_d \to K^0\bar{K}^0\right) \\
&\simeq 24.65 \times 10^{-6},
\end{aligned} \tag{21}$$

where we have taken $Br\left(B_d \to K^0\bar{K}^0\right) \simeq 1.33 \times 10^{-6}$ [24]. We note that the decay rate of $B_s \to K^0\bar{K}^0$ can also be estimated based on the $SU(3)$ symmetry with the breaking effect and neglecting the small annihilation contribution. Explicitly, we have

$$\begin{aligned} Br(B_s \to K^0\bar{K}^0) &\simeq \frac{\tau_{B_s}}{\tau_{B_u}}\left(\frac{M_{B_s}}{M_{B_u}}\right)^3 \left|\frac{F^{B_sK}(0)}{F^{B_u\pi}(0)}\right|^2 Br\left(B_u^{\pm} \to \pi^{\pm}K^0\right) \\ &\simeq 24.19 \times 10^{-6}\,, \end{aligned} \tag{22}$$

which agrees well with Eq. (21), where we have used $Br\left(B_u^- \to \pi^- K^0\right) \simeq 21.0 \times 10^{-6}$ [15, 16, 17]. From the estimations, we find that $Br\left(B_s \to K^+K^-\right)$ prefers to being less than $Br\left(B_s \to K^0\bar{K}^0\right)$. It is clear that with specific values of relevant physical observables, Eqs. (18), (19) and (20) can fix γ. To illustrate our results, in Figure 2, we plot Eq. (20) in the $\xi - \gamma$ plane with $R_B \simeq 0.91$ as an input value and we find that γ is about 73^0. From Eq. (13), we see that the sign associated with $\cos\delta\cos\gamma$ is positive. Since $\left|V_u\right|^2 R^2/\left|V_c\right|^2 \sim \lambda^4 R_b^2 R^2$ which is in a few percent level and negligible, once $R_B < 1$ (> 1) is measured, one can conclude that $\cos\delta\cos\gamma < 0$ (> 0). Moreover, through the $\cos\delta$ described by Eq. (15), one can also obtain the information whether γ is larger or less than 90^0. In particular, for $R_B \simeq 1$ while $\cos\delta\cos\gamma \simeq 0$, one gets $\gamma \simeq 90^0$ from Eq. (15). By fixing the relevant observables except R_B, in Figures 3 and 4, we show how the angle of γ is sensitive to the value of R_B.

It is worth to mention that the interference term in Eq. (5), associated with $\cos\delta'\cos\gamma$ for $Br\left(B_d \to \pi^+\pi^-\right)$, is negative. Therefore, in contrast to the situation in the decay of $B_s \to K^+K^-$, the BR of $B_d \to \pi^+\pi^-$ will have larger (smaller) values if $\cos\delta'\cos\gamma < (>)0$. From Eq. (5) one can evaluate the BR of $B_d \to \pi^+\pi^-$ model-independently and the results are shown in Table 1.

Table 1: The BR (in units of 10^{-6}) of $B \to \pi^+\pi^-$ with $R(r) = 8.0$, $\delta^{(\prime)} = 220^0$ and (I) $\gamma = 70^0$, (II) $\gamma = 90^0$, and (III) $\gamma = 110^0$.

BR	model-independent	experiment
I	5.66	$4.1 \pm 1.0 \pm 0.7$[15]
II	4.87	$5.9^{+2.4}_{-2.1} \pm 0.5$[16]
III	4.08	$4.3^{+1.6}_{-1.5} \pm 0.5$[17]

Unfortunately, since the current accuracy of experiments is limited, at the moment, we still cannot determine whether $\cos\delta'\cos\gamma$ is positive or negative but it can be done in future B facilities. For a comparison, we display the BRs of $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$ as a function of γ in Figure 5. From the figure, it is interesting to find out that one of the distributions increases with respect to the angle γ, while the other one decreases.

In summary, we have studied the possibility of determining γ by using the virtual measurements of $A_{CP}^{mix}\left(B_d \to \pi^+\pi^-\right)$, $A_{CP}^{dir}\left(B_d \to \pi^+\pi^-\right)$, β, R_{CP} and R_B. The first three physical quantities can be measured precisely in the e^+e^- machines at the $\Upsilon(4S)$ resonance as well as hadronic ones such as the Tevatron Run II. However, R_{CP} and R_B can be only observed in the hadronic machines. It is known that the Tevatron Run II has started a new physics run at $\sqrt{s} = 2$ TeV and will collect a data sample of 2 fb^{-1} in the first two years [25]. At its initial phase with 10K of $B_s \to KK$ events and by the relevant measurements in $B_d \to \pi^+\pi^-$ decays, the observed R_B could provide us a good opportunity to determine γ.

Acknowledgments

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Figure Captions

Figure 1: ξ as function of γ with the values of $A_{CP}^{mix}(B_d \to \pi^+\pi^-) = 0.45$, $A_{CP}^{dir}(B_d \to \pi^+\pi^-) = -23\%$, $R_{CP} = 1.4$ and $sin2\beta = 0.60$. The solid and dashed lines correspond to Eqs. (18) and (19), respectively.

Figure 2: Same as Figure 1 but including Eq. (20) (dash-dotted line) with $R_B = 0.91$.

Figure 3: ξ as a function of γ with $A_{CP}^{mix}(B_d \to \pi^+\pi^-) = 0.1$, $A_{CP}^{dir}(B_d \to \pi^+\pi^-) = -28\%$, $R_{CP} = 1.8$, $sin2\beta = 0.45$ and $R_B = 1.02$. The curves are labeled as in Figure 2.

Figure 4: Same as Figure 3 but with $R_B = 1.18$.

Figure 5: BRs (in units of 10^{-6}) of $B_s \to K^+K^-$ (dashed line) and $B_d \to \pi^+\pi^-$ (solid line) as a function of γ with R(r)=8.0 and (a) $\delta = 220^0$ and (b) $\delta = 40^0$.

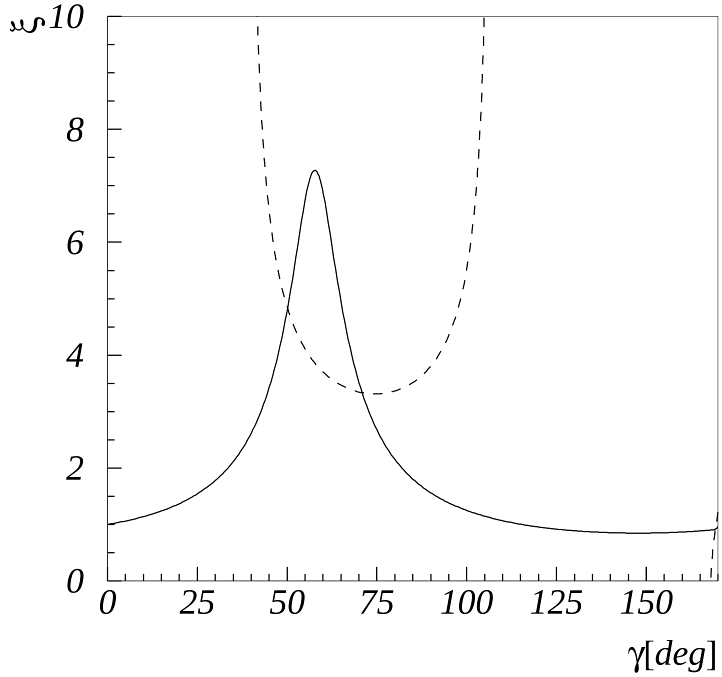


Figure 1: ξ as function of γ with the values of $A_{CP}^{mix}(B_d \to \pi^+\pi^-) = 0.45$, $A_{CP}^{dir}(B_d \to \pi^+\pi^-) = -23\%$, $R_{CP} = 1.4$ and $sin2\beta = 0.60$. The solid and dashed lines correspond to Eqs. (18) and (19), respectively.

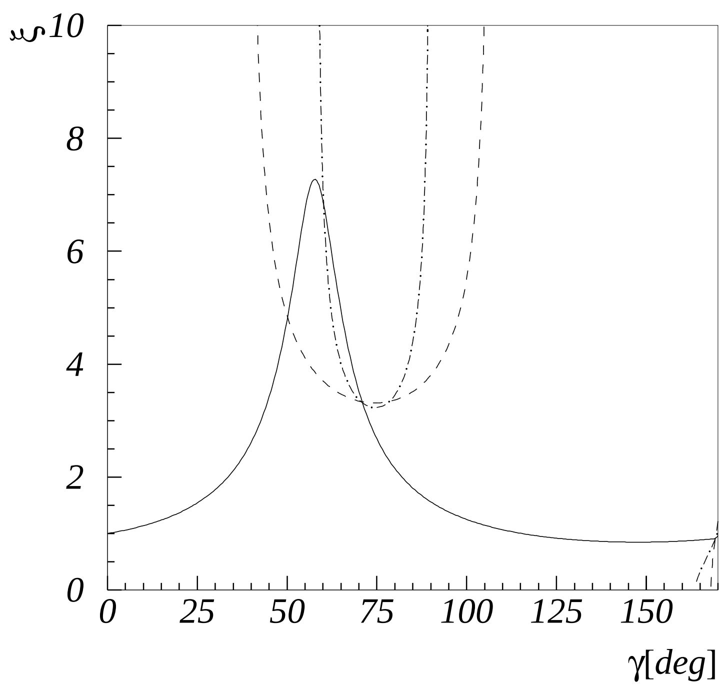


Figure 2: Same as Figure 1 but including Eq. (20) (dash-dotted line) with $R_B = 0.91$.

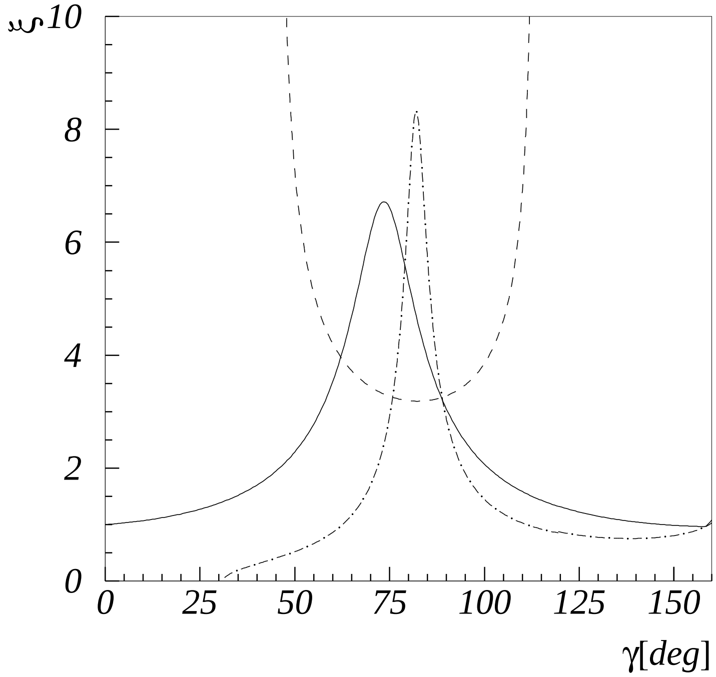


Figure 3: ξ as a function of γ with $A_{CP}^{mix}(B_d \to \pi^+\pi^-) = 0.1$, $A_{CP}^{dir}(B_d \to \pi^+\pi^-) = -28\%$, $R_{CP} = 1.8$, $sin2\beta = 0.45$ and $R_B = 1.02$. The curves are labeled as in Figure 2.

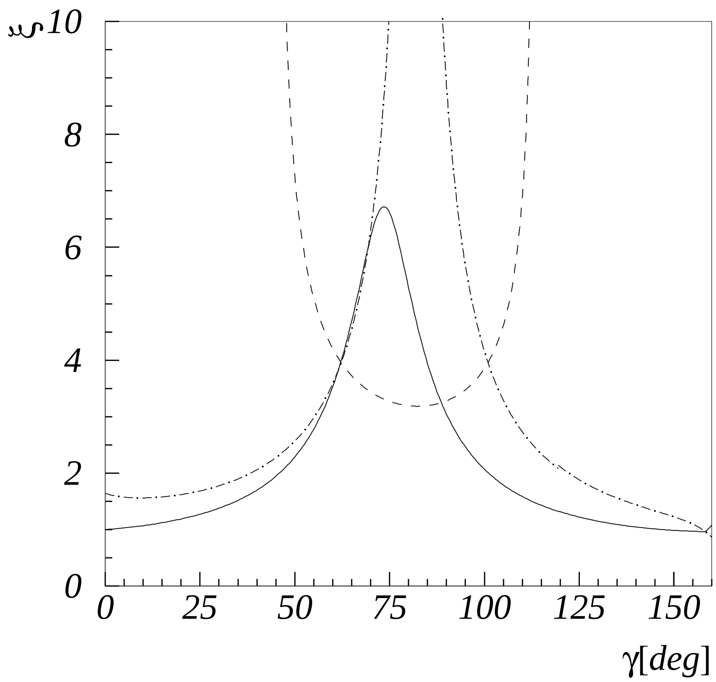
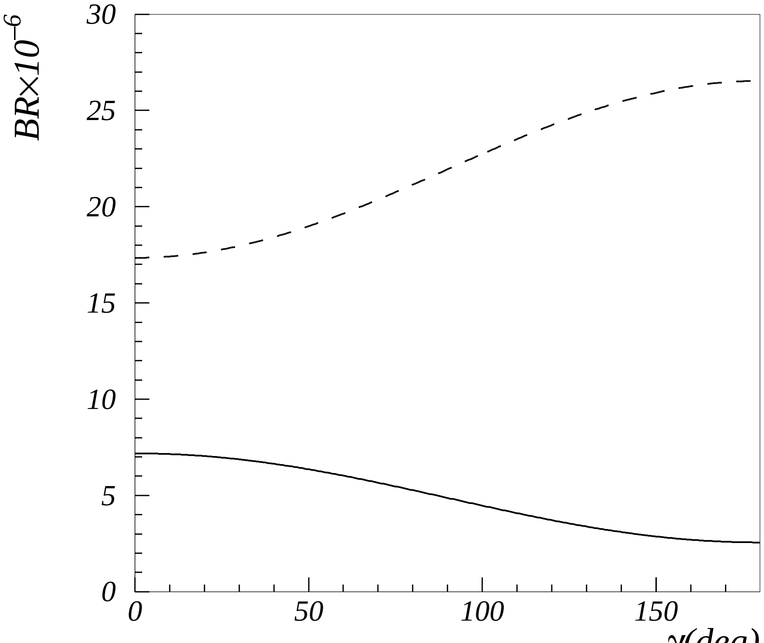
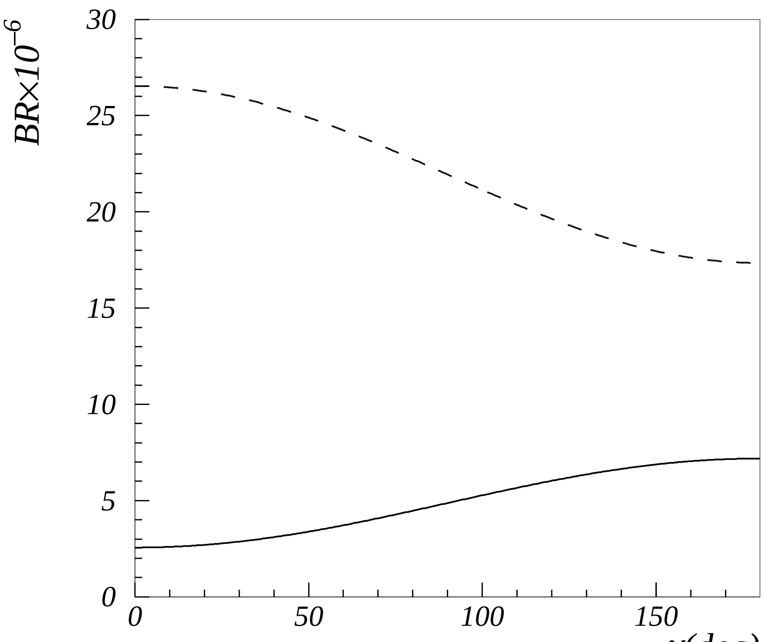


Figure 4: Same as Figure 3 but with $R_B = 1.18$.



(a)



(b)

Figure 5: BRs (in units of 10^{-6}) of $B_s \to K^+K^-$ (dashed line) and $B_d \to \pi^+\pi^-$ (solid line) as a function of γ with R(r)=8.0 and (a) $\delta = 220^0$ and (b) $\delta = 40^0$.